

We begin with a question:

Q: What is the nullspace of an invertible matrix?

Q: What is the nullspace of $\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$?

Solution.

We need that

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

so $x + y = 0$ which gives us that $x = -y$. Hence, the nullspace of A is one-dimensional and spanned by $(1, -1)^T$.

Q: What is the nullspace of

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}?$$

Solution.

The nullspace of the derivative is the space of constant functions.

Exercise. What can one say about $\text{Null}(A)$, $\text{Null}(B)$, and $\text{Null}(AB)$?